GAMBREL ROOF ARCHES FOR FARM AND STORAGE BUILDINGS

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A computer program was developed to design a family of traditionally shaped gambrel roof 3-hinged arches for low human occupancy farm buildings. Upper and lower rafters were sized for minimum waste using stock dimension lumber, spruce or Douglas fir, spaced 600 mm o.c., giving clear spans from 6 to 21.6 m. The program sets rafter lengths and slopes within specified slope limits, and tabulates for each nominal rafter size, specie and grade, the limiting safe design snow and wind loads. Designs are based on rafter moments and axial forces resulting from climatic loads combined with dead loads according to the National Building Code of Canada (1977). Connections are nailed plywood gussets similar to Canada Plan Service trusses. The complete family of arches are now detailed in 52 plan sheets, complete with tabulated dimensions, member sizes, nailing specifications, safe climatic loads and foundation reaction forces.

INTRODUCTION

The gambrel roof (sometimes incorrectly called 'hip' roof) is a shape associated with the traditional North American barn. This roof has retained its popularity with farmers and builders for several reasons. Framed as four rafters connected to make an arch approaching the proportions of a parabola, it is aesthetically pleasing, it provides a clear span for storage space uninterrupted by columns, and being framed with the absolute minimum number of planks it is structurally efficient.

In 1975 the Canada Plan Service (CPS) committee requested an engineering study on the gambrel roof arch. More and more farmers, including those with riding horses, were asking for barn plans styled with the traditional gambrel roof. Almost no engineering work has been done on this, probably because most of the locally-made braced rafter gambrel roofs have survived decades of wind and snow without damage, and without benefit of engineering analysis. Now, however, some provinces require farm structures to be built according to a building code; as a result, authorities require engineered drawings that can be approved in advance and checked against actual construction.

The authors anticipate a wide variety of uses for gambrel roof arch designs. These uses include conventional two-story barn roofs as well as single-storey storages for produce, grains, and farm machinery. For bulk storage, the sloping lower rafter retains a pile of granular material at an angle approaching its natural angle of repose, thus minimizing the lateral pressure. In contrast, vertical walls must be very heavily reinforced to contain bulk granular materials such as potatoes and grains.

Manufacturers of prefabricated frameless steel arches seldom publish safe climatic loads for their structures, probably because analysis of no-hinged arches is more difficult and less predictable. In contrast, the gambrel roof can be easily analyzed as a statistically-determinate 3-hinged arch. And wood braced rafters, spaced at modular 400- or 600-mm centers, are easier to insulate, vapor-seal and finish with inexpensive conventional building materials.

About 1955 a series of 'rigid frame' 3-hinged arch designs was introduced in Canada, USA and Europe, by the (then)
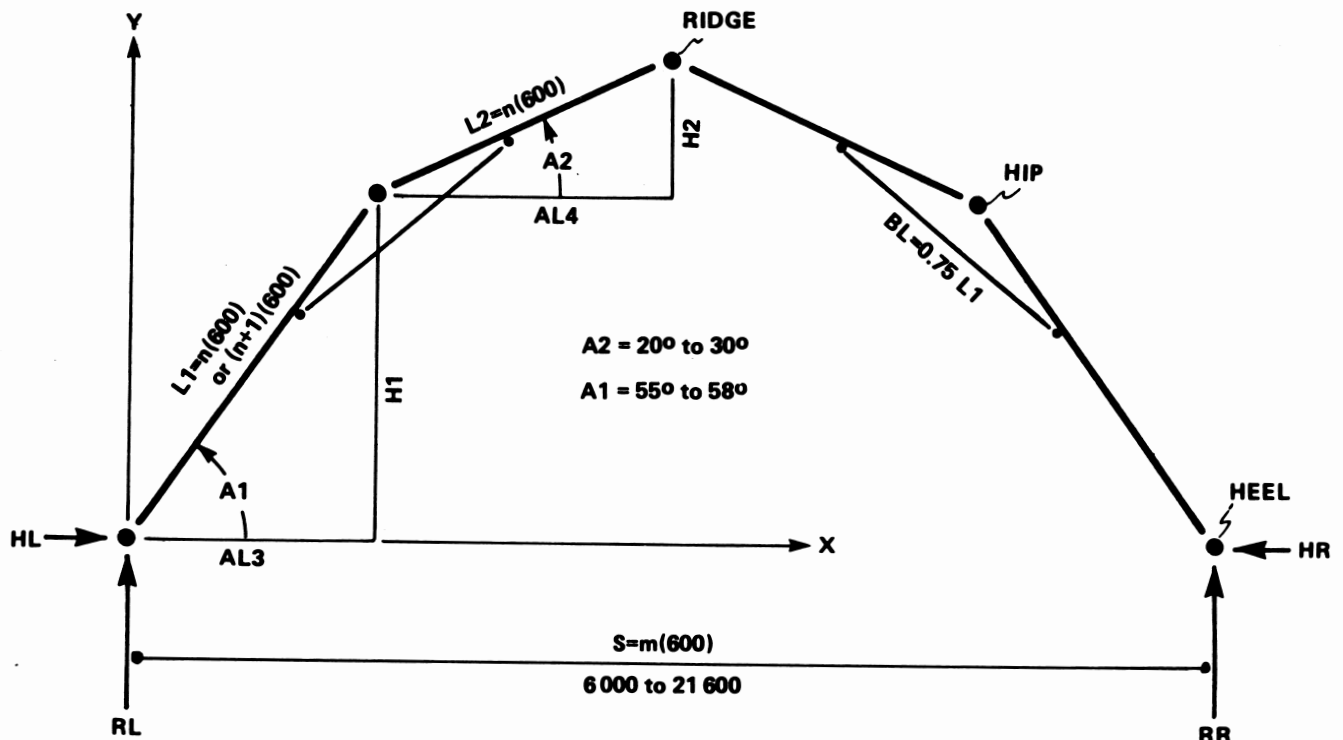

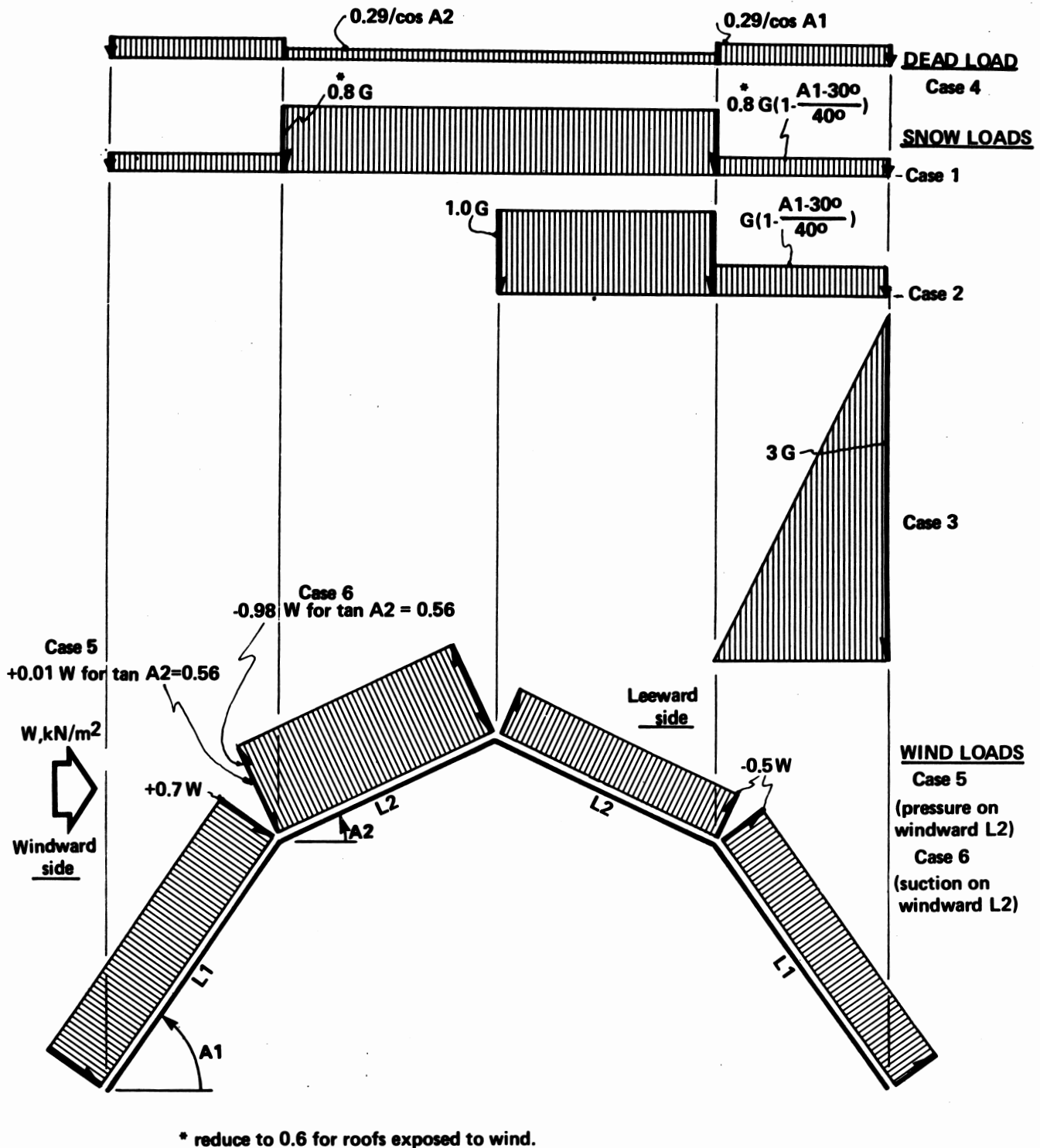**Figure 1.** Braced rafter arch geometry.

Figure 2. Dead, snow and wind loads used for design of braced rafters.

Plywood Manufacturers of British Columbia (PMBC). Payne (1971) later republished the design concepts. These PMBC rigid frames were very popular for a time. The designs used short lower rafters and much longer upper rafters, which for wider building spans required sawn lumber over 4.8 m long. Also, bending moments in the long upper rafters became excessive.

The objective of this study was therefore to investigate the proportions and structural efficiency of the traditional gambrel roof shape, to develop 3-hinged arch designs to optimize the use of readily available sawn lumber sizes, and to publish the designs in a form easily used by farmers, rural building

contractors and building officials.

DESIGN CONCEPTS — GAMBREL ROOF ARCHES

Arch Form

Preliminary calculations showed that a gambrel roof arch form could be adjusted

within broad limits to safely maximize useful storage volume while minimizing building material costs. This optimization yielded the arch form shown in Fig. 1, with lower rafter angles A1 set between 55° and 60°, and upper rafter angles A2 between 20° and 30°. To ensure the use of stock lumber lengths for minimum cutting waste, all rafter lengths were set in multiples of 600 mm. Rafter lengths were either all equal (L1 = L2), or the lower rafters (L1) were 600 mm longer than the upper ones (L2). The lengths of hip joint brace members (BL) were set at 0.75 times the lower rafter length L1, truncated down to the nearest whole 300-mm length, and located symmetrically about the hip joint. All joints (heel, hip, ridge and brace ends) were considered as hinged, making the arch statically determinate, and no adjustments were made to allow for possible stiffness or strength increases due to the semi-rigid nailed connections actually used.

Design Loads

Internal storage loads were not considered, but external climatic loads were taken mostly from the National Building Code of Canada (1977), as detailed in Supplement No. 4. As usual for farm buildings, earthquake loads were not considered. Snow loads were considered as illustrated in Fig. 2, cases 1, 2 and 3; each snow load was added to dead load, case 4. Dead load was assumed at 0.29 kN/m² of roof surface, considered in the plane of the roof; this was divided by the cosines of roof angles A1 and A2 respectively, to convert to roof surface horizontally projected. Snow load cases 1 and 2 were taken directly from the Code (1977); case 3 was developed to check for the single-storey situation where the arch heel is supported very close to grade. In this case, a combination of drift snow and snow sliding off the upper rafters can pile against the lower rafter L1. Taylor (Personal communication; D.A. Taylor, Structures Section, Div. of Bldg. Res., Nat. Res. Council, Montreal Road, Ottawa, Ont. K1A 0R6) was consulted for the distribution and magnitude of that portion of the snow pile estimated to be supported by the roof (up to 3 G in this case).

Wind loads were based partly on the National Building Code of Canada (1977) which does not give pressure coefficients specifically for gambrel roofs, and partly on Fenton and Otis (1941). For the lower windward rafter, a pressure coefficient of + 0.7 was taken from Fenton and Otis (1941); for the upper windward rafter, the Code suggests checking a range from positive through negative pressures, as shown in Fig. 2, cases 5 and 6. For the leeward roof slopes, -0.5 is given by both references. For each arch form and rafter size selected, pressure coefficients were used to determine maximum allowable wind pressures. These pressures in turn were related to the maximum 1-h wind speed having one chance in 10 of being exceeded in any given year,

multiplied by 2.0 (gust factor) and 1.0 (exposure factor).

Connections and Sawn Lumber

It was decided to use the same type of connections as in the CPS trusses; that is, 12-mm Douglas fir plywood gussets nailed to both sides of 38-mm spruce or Douglas fir frame members. Special 5 x 64-mm (6 gage X 2½ inch) 'Truss Gusset' nails are not always available (or suppliers won't bother to order them), so 4 x 64-mm special concrete nails were substituted. Like the 'Truss Gusset' nails formerly specified, these develop two shear planes (double shear) by fully penetrating two plywood gussets when nailed from both sides of the joint. Based on Turnbull and Theakston (1964) basic double shear nail loads of 778 and 970 N/nail for spruce and Douglas fir respectively were used, modified by the 1.25 'low human occupancy' factor allowed by the Canadian Farm Building Code (1977) as well as 'load duration' factors appropriate to the climatic loads considered (1.15 for snow, 1.33 for wind) as listed in CSA Standard 086 (1976). 'Dry' service conditions were assumed (factor 1.00); whenever arches could be exposed to 'wet' service such as in uninsulated winter animal housing, it is suggested that safe climatic loads determined for a given arch design should be reduced by a 0.75 multiplying factor. This factor was set between the 0.84 factor for wood members in bending and the 0.69 for compression parallel to grain, as recommended by CSA 086 (1976). This is because braced rafters are typically stressed by bending and axial compression combined.

PROGRAM METHODOLOGY

The major computer program steps are listed as follows:
1. Input rafter size, rafter span, low or high human occupancy, rafter spacing, lumber grade and type.
2. Calculate rafter lengths, slopes and brace length.
3. Determine snow, wind and dead load coefficients based on rafter slopes.
4. Calculate reactions, moments, axial and shear forces due to unit snow, unit wind and dead loads.
5. Determine allowable compression stress in rafter.
6. Determine location of critical combined axial and bending stress, for load cases 1, 2, 3, 5 or 6, each added to dead load 4.
7. Determine maximum allowable ground snow load G for cases 1, 2 or 3, each added to 4.
8. Determine maximum allowable 1/10 hourly wind load for cases 5 or 6, each added to 4.
9. Check maximum allowable 1/10 hourly wind load when combined with maximum allowable ground snow plus dead load (from step 7).
10. Check maximum allowable ground

snow load when combined with maximum 1/10 hourly wind plus dead load (from step 8).
11. Determine critical force at ridge, brace, hip and heel.
12. Determine number of nails at ridge, brace and hip gussets.
13. Check compression stresses in 38 x 89-mm brace.
14. Print results.

The assumptions for steps 2, 3 and 12 above have already been explained. In step 4 the moment, axial and shear forces were calculated at 25-mm horizontal increments across the span of the arch for each load case. The axial force in the brace on each side is calculated for each load case.

In step 5, the program assumes that all rafters are laterally supported in the thickness dimension by roof purlins in order to qualify as 'short columns' for assigning allowable compression stresses. The program checks the slenderness ratio in the depth direction and modifies the allowable compressive stress for this if necessary. End fixity factor $k = 0.65$ was assumed.

In step 6, the trial and error method is used to find the location of the critical combined stress.

In steps 7, 8, 9 and 10, trial and error iterations are used to determine the loads.

In step 13, the program checks the slenderness ratio of the brace in both the thickness and depth directions of the member (assumed $k = 0.65$) as before. In depth, the compressive stress is reduced if necessary; in thickness, lateral bracing and/or compressive stress reduction are applied.

RESULTS AND DISCUSSION

Figures 3 and 4 illustrate the relative magnitudes of bending moments and axial forces, plotted for a specific arch (12.6-m span, 38 x 235-mm no. 2 spruce rafters) at maximum allowable combined stress. In Fig. 3, cases 2 + 4 (unsymmetrical snow + dead) were critical. These represented a ground snow load of 2.13 kN/m², exposed windy location (corresponding roughly to Bowmanville, Ontario). Cases 5 + 4 were critical for wind, representing a 1/10 hourly wind pressure of 0.61 kN/m² (corresponding to Lethbridge, Alberta).

The computer program (step 6, Program Methodology) combines the effects of moments and axial forces in each rafter. Of these, the moment component (Fig. 3) of the combined stress was the major one, whereas the axial force component (Fig. 4) was rather small. The following relation was used to check combined stress effects:

$$\frac{M}{SF_b} + \frac{P}{AF_c} \leqslant 1.0 \dots\dots\dots\dots\dots (1)$$

where

M = bending moment, N.m

S = rectangular section modulus $= \dfrac{bd^2}{6}$

b = rafter thickness, mm
d = rafter depth, mm
F_b = allowable bending stress, MPa
P = axial force, N
A = area of section, (b·d), mm^2
F_c = allowable axial stress, MPa

For example, a critical moment showed in case 2 (unsymmetrical snow drift load) in the upper leeward rafter (see Figs. 3 and 4). The following example calculation gives the mathematics used to evaluate the combined bending + axial stresses:

	Drift snow (case 2)	Dead load (case 4)		
Critical moment, M	2987 +	272	=	3267 N.m
Axial force, P	(−2487) +	(−903)	=	−3390 N

Substituting into equation 1:

$$\frac{M}{SF_b} + \frac{P}{AF_c} = \frac{3267\ (100)}{(350\ 530)\ (9.804)} +$$

$$\frac{3390}{(38 \times 235)\ (8.22)}$$

$$= 0.950 + 0.046$$

$$= 0.996 < 1.0$$

In the above case, 95% of the stress in the extreme fiber was due to bending and only 5% was due to axial force. Axial forces shown in Fig. 4 are therefore not very important here. Note that Figs. 3 and 4 are not directly comparable in scale, since they are in different dimensions.

It is also interesting to note in Fig. 3 that unsymmetrical loadings (drift snow, case 2, and wind, case 5) are more critical than symmetrical loadings. This critical loading is typical of arch-type buildings in general, and is supported by case histories of documented arch roof failures.

Regarding the possible simultaneous occurance of dead load + maximum snow + maximum wind, the National Building Code (1977) requires the summing of the three effects but adjusted by a factor of 0.75. In this case the combination of dead, wind and snow loads results in wind uplift on the upper leeward rafter partly cancelling the snow + dead load effects (see Fig. 3, cases 1, 2 and 6). The possibility exists for wind to deposit snow on the leeward roof (case 2) then reverse direction 180° after the snow drift has frozen in place, but the authors reasoned that this possibility was too remote to be considered.

A comparison with the old rigid frame designs published by the PMBC (1965) is interesting. The improved gambrel-roof braced rafter given in the above example uses 38 x 235-mm lumber for both upper and lower rafters, and the longest rafter piece is 4.2 m long. The corresponding rigid frame required doubled 38 x 286-mm rafters each 6.6 m long. This indicates that the objective of this project was achieved; that is, to
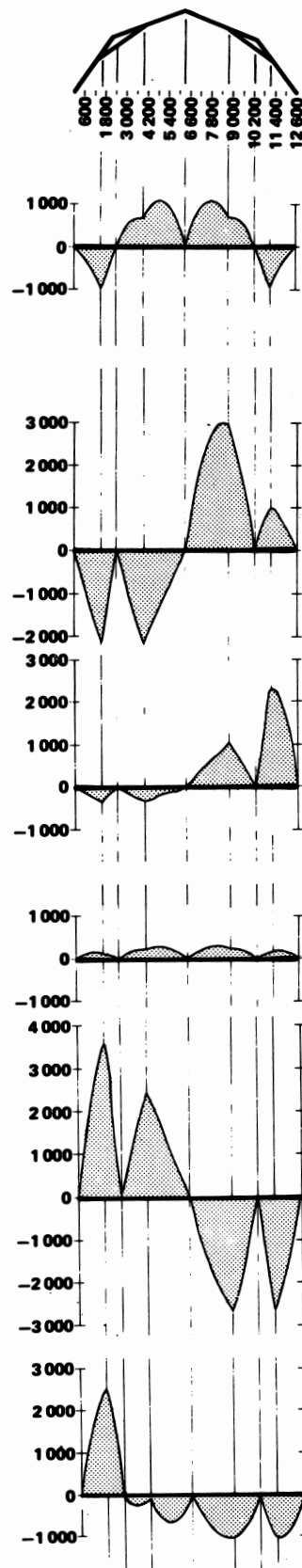

Figure 3. Moment diagram for a gambrel roof braced rafter, span 12.6 m, spacing 600 mm o.c., rafter size 38 x 235 mm, moments scaled in N.m.
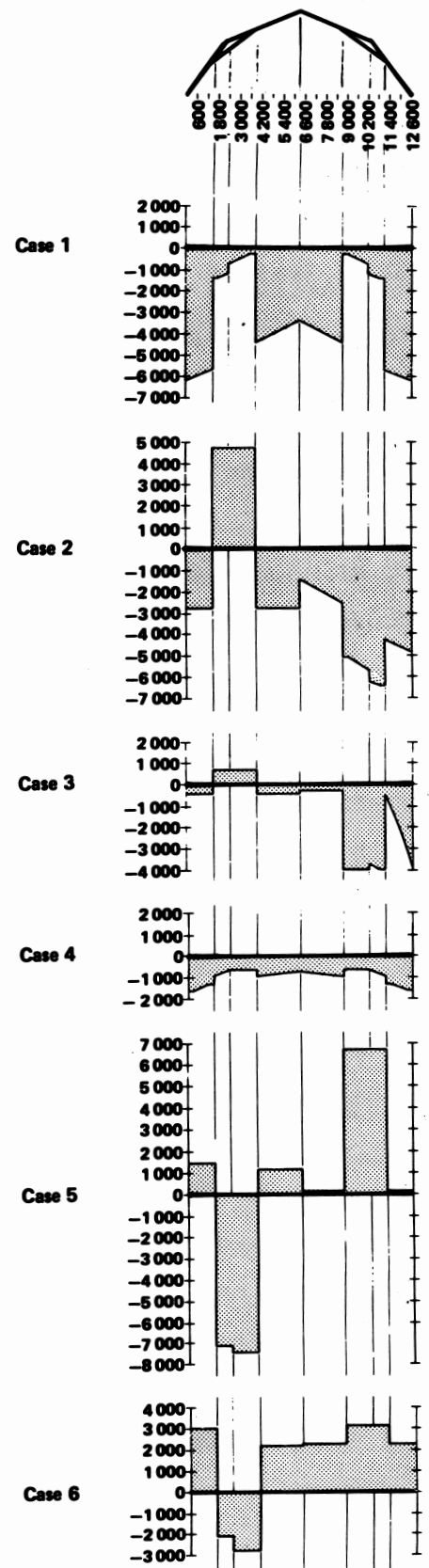

Figure 4. Axial force diagrams for gambrel roof braced rafter, span 12.6 m, spacing 600 mm o.c., rafter size 38 x 235 mm, forces scaled in N.

improve the structural efficiency of wood arches.

SUMMARY

The Canada Plan Service has written a computer program for design of braced rafter farm building gambrel roofs analyzed as 3-hinged arches. The concept on which the roof shape is based is historical, but the engineered result is a series of arch designs which are much more efficient than older designs, especially with the smaller lumber sizes now available in Canada. These designs are published in the CPS Building Engineering Series, in a format suitable for direct use by extension engineers, builders and farmers.

Connections are based on the traditional Canada Plan Service method, using large, hardened steel nails driven to penetrate plywood gussets on both sides of the frames and resulting in double-shear loading of each nail. Readily-available hardened concrete nails were used in preference to the special Truss Gusset nails formerly used in CPS trusses.

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